Proc. IAHS, 372, 399–401, 2015
proc-iahs.net/372/399/2015/
doi:10.5194/piahs-372-399-2015

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Research on the deformation of a confined aquifer based on Cosserat continuum mechanics

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Published: 12 November 2015

Abstract. Recent monitoring of land subsidence and soil deformation indicates a new phenomenon where excessive and continuous deformation occurs in the sandy aquifers in Shanghai and the Su-Xi-Chang region of China. It is hard to explain factors contributing to this phenomenon with traditional Cauchy continuum mechanics in which low normal stress in the ground could not cause such large deformation. Steep hydraulic gradient would be formed in the aquifer if groundwater is pumped from densely distributed wells, and shear stresses would develop then. Accumulated shear stress could then lead to deformation of the aquifer or even land subsidence. Accumulated shear stress due to the drawdown of groundwater level is one of the main factors that contribute to deformation within an aquifer. Traditional Cauchy continuum mechanics cannot consider this shear stress because of the hypothesis of equal shear stress in the aquifer unit. Cosserat continuum mechanics can be applied to analyse the mechanism of aquifer deformation controlled by accumulated shear stress by considering the scale effect and the asymmetric distribution of shear stress in the aquifer unit.

1 Introduction

Land subsidence due to groundwater exploitation is of global concern. Recent research presents some new characteristics of aquifer deformation including excessive deformation which occurs in aquifer systems, and delayed deformation compared with groundwater decline in coastal areas in China (e.g. in Shanghai and the Su-Xi-Chang region). These new characteristics are significant for investigating the continuing deformation of sandy aquifers (Gong et al., 2009; Wang et al., 2010; Zhang et al., 2006). Some researchers have discussed factors contributing to such deformation in terms of aquifer properties (Wang et al., 2010; Zhang et al., 2006), however the yield mechanism of the sandy aquifer under lower vertical stress has not previously been addressed.

The hydraulic gradient due to groundwater withdrawal can contribute to shear stress development in a sandy aquifer. Some research demonstrates that the sand gets to a critical state under the high shear stress level (Cai, 2004). The accumulated shear stress due to the hydraulic gradient can result in the deformation of the aquifer or even land subsidence. Therefore, the accumulated shear stress in the aquifer may be one of the main factors contributing to the deformation of the aquifer sand. Traditional methods based on Cauchy mechanics does not consider the shear and rotational deformation caused by accumulated shear stress (Budhu and Adiyaman, 2010). It is therefore important to investigate a new prediction model which considers the asymmetric distribution of shear stress in aquifers.

2 Budhu's approach based on Cosserat continuum mechanics

Cosserat continuum mechanics was developed to analyse the deformation of a continuous medium. It simultaneously considers the isotropic effective stress and the torsion on the unit cell in the equilibrium equation. Budhu and Adiyaman (2010) proposed an approach to predict land subsidence in an unconfined aquifer based on Cosserat mechanics. Figure 1 shows the stress distribution in an unconfined aquifer with a variation in groundwater level. Based on Budhu's approach, the changes in effective stress due to the decreasing groundwater level are broken down into two parts. Part I is an

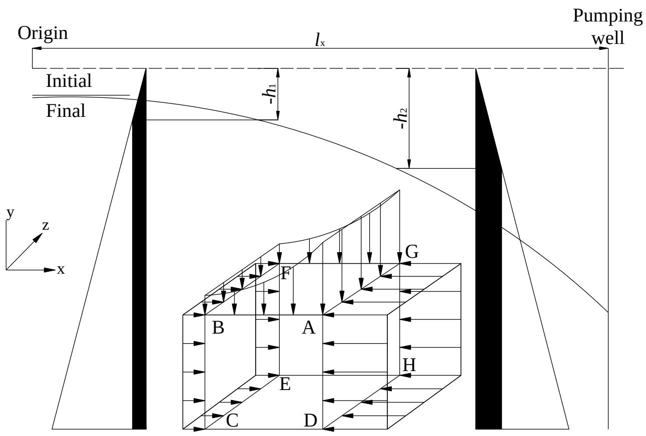

Figure 1. Stress distribution with a change in groundwater level (Budhu and Adiyaman, 2010).

isotropic effective stress, which contributes to isotropic compression. Part II is a vertical effective stress varying with lateral distance, which causes simple shear and rotation. The isotropic deformation due to compression stress is analysed using Cauchy mechanics, and the deformation due to the shear stress and torsion is simulated using Cosserat continuum mechanics. The hypotheses of Budhu's approach are as follows: (1) the aquifer is an isotropic elastic continuous medium without any cracks; (2) there is no replenishment from other aquifers to the unconfined aquifer. The stress in the unconfined aquifer remains unchanged when the groundwater level changes; (3) the groundwater level in the initial state is the same in the horizontal direction; (4) the effective stress in the aquifer changes instantaneously when the groundwater level changes.

3 Shen's approach based on Cosserat continuum mechanics

3.1 Hypotheses for Shen's approach

Budhu's approach is not appropriate for confined aquifers because it was proposed based on the unconfined aquifers in Arizona. In order to apply Budhu's approach in confined aquifers, the following additional hypotheses for confined aquifers (Shen et al., 2013) are presented: (1) the confined aquifer is an isotropic, uniform continuous medium without cracks; (2) the overburdened load from an aquitard above an aquifer remains unchanged during groundwater pumping from the aquifer; (3) variation in groundwater replenishment from the overburdened aquitard depends on the consolation rate of the overburdened aquitard; (4) the groundwater level in the initial state remains the same in the horizontal direction; (5) the groundwater level in the aquifer declines slowly with pumping time.

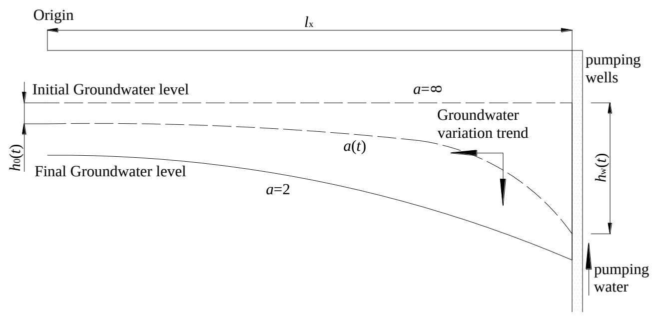

Figure 2. Variation tendency of groundwater level in aquifer.

3.2 Shen et al. (2013)'s equation

To simulate the rate of the replenishment from upper overburdened aquitards, μ' is introduced, and is included in Shen et al. (2013)'s equation as follows:

$$\overline{\sigma}'_{ij} = \mu' \left(\frac{1}{V}\right) \int_V \sigma'_{ij} \mathrm{d}V, \tag{1}$$

where σ'_{ij} is the effective stress tensor, $\overline{\sigma}'_{ij}$ is the average effective stress tensor, and V is the volume of the soil unit. At the start of pumping, when there is a large hydraulic gradient between the aquifer and the aquitard, groundwater replenishment is high and the effective stress in the aquifer increases slowly so that μ' equals 0. After a period of time, once groundwater motion becomes a stable seepage and the soil completes consolidation in the aquitard, the effective stress in the aquifer reaches its maximum value; in this case, μ' equals 1.

Figure 2 shows the variation tendency of the groundwater level in the aquifer. The groundwater in the initial state is assumed to be in the horizontal direction. When the groundwater is pumped from the aquifer, the groundwater level declines gradually with time. To consider the impact of time on groundwater level, the groundwater level equation is as follows:

$$h_\mathrm{w}(t) = -\left(\alpha_i x^a + h_0(t)\right), \tag{2}$$

where $h_\mathrm{w}(t)$ is the drawdown of the groundwater head with time from the initial state at the pumping well, $h_0(t)$ is the drawdown of the groundwater head with time from the initial state at an arbitrary selected origin, α_i is a soil layer constant, x is the distance from the origin, and a is the time factor, which can be calculated using the following equation:

$$a = \lambda' - \ln t \tag{3}$$

where t is time, and λ' is a constant time factor.

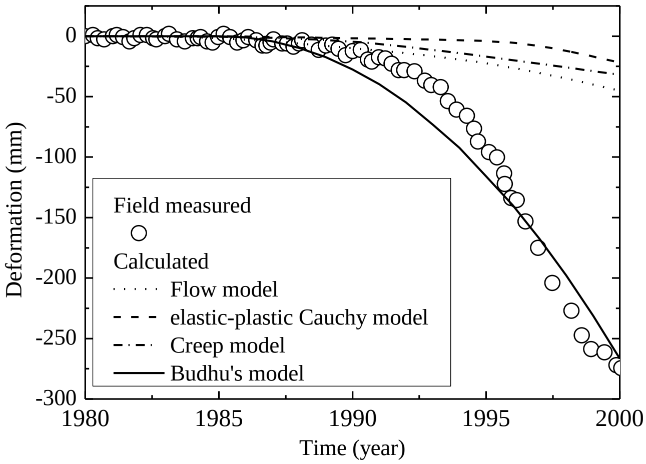

Figure 3. Comparison of the deformation of Aquifer IV at Laodong Park.

4 Discussion

Shen et al. (2013) used the modified approach to simulate the deformation of Aquifer IV in Shanghai. Figure 3 depicts the measured and calculated curves using four different models: (1) groundwater flow model, (2) elasto-plastic model based on Cauchy mechanics, (3) model considering the creep behaviour of sandy soil, and (4) Shen et al. (2013)'s approach based on Cosserat mechanics. The results predicted by the classic models (groundwater flow model, elasto-plastic model, and creep model) do not fit the measured data well. However, Shen et al. (2013)'s approach based on Cosserat mechanics can reasonably predict the variation in deformation of Aquifer IV in Shanghai.

5 Conclusions

1. Budhu's approach based on Cosserat mechanics can be used to predict the deformation of an unconfined aquifer. As the consolidation effect of the overburdened aquitards is not considered, Budhu's approach cannot directly predict the deformation of a confined aquifer.

2. Shen et al. (2013) modified Budhu's approach to simulate the deformation of a confined aquifer by considering the effect of an overburdened aquitard and the impact of time on groundwater level.

Acknowledgements. The research work described herein was funded by the National Nature Science Foundation of China (Grant No. 41472252) and partially funded by the National Basic Research Program of China (973 Program: 2015CB057806). This financial support is gratefully acknowledged.

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