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A new empirical model for estimating the hydraulic conductivity of low permeability media

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Abstract Hydraulic conductivity (*K*) is one of the significant soil characteristics in terms of flow movement and solute transport. It has been recognized that *K* is statistically related to the grain-size distribution. Numerous models have been developed to reveal the relationship between *K* and the grain-size distribution of soil, but most of these are inappropriate for fine-grained media. Therefore, a new empirical model for estimating *K* of low permeability media was proposed in this study. In total, the values of *K* of 30 soil samples collected in the Jiangning District of Nanjing were measured using the single-ring infiltrometer method. The new model was developed using the percentages of sand, silt and clay-sized particles, and the first and the second rank moment of the grain-size through the moment method as predictor variables. Multivariate nonlinear regression analysis yielded a coefficient of determination (R^2) of 0.75, indicating that this empirical model seems to provide a new approach for the indirect determination of hydraulic conductivity of low permeability media.

Key words hydraulic conductivity; empirical model; grain-size analysis; fine medium

1 INTRODUCTION

The hydraulic conductivity (*K*) of soil is of great significance in hydrogeology. All the development, management and protection of groundwater and the prediction of contaminant transport need reliable estimates of *K*.

Hydrogeologists have searched for reliable techniques to determine the *K* of soils, for better groundwater development, management and conservation. Many different techniques have been presented, including field methods, laboratory methods and empirical formulae. However, precise estimation of *K* by field techniques is limited by the lack of accurate information of the aquifer geometry and hydraulic boundaries and always prohibited by the high cost for the construction of observation wells (Chen *et al.* 2010). Laboratory tests, on the other hand, present formidable problems in the sense of obtaining representative samples. It has long been recognized that *K* is statistically related to the grain-size distribution of granular porous media. As a result, numerous models estimating *K* from empirical formulae based on grain-size distribution, have been developed and used to overcome these problems. Grain-size methods are less expensive and do not depend on the geometry and hydraulic boundaries of the aquifer. Most significantly, information about the textural properties of soils is easy to obtain. Consequently, groundwater professionals have tried for decades to reveal the correlation between *K* and the grain-size distribution of soil. The tasks appear rather straight forward, but it is found that this correlation is not easily established (Pinder and Celia 2006).

So far, many predictive methods have been revealed to estimate the *K* from grain-size analysis which use the analogy of pipe flow and flow in capillaries to represent water flow in soil (Kozeny 1927; Carmen 1937, 1956). Besides these methods, empirical formulae have also been developed (Hazen 1892; Krumbein and Monk 1942). A popularly accepted relationship was proposed by Hazen (1892) with the form:

$$K = Ad_{10}^2 \tag{1}$$

where K is hydraulic conductivity (cm/s); A is a constant; and d_{10} is the effective diameter. In order to explain the distribution of the grain-size curve, Masch and Denny (1966) replaced the effective diameter with the median grain size d_{50} as the representative size in an attempt to correlate *K* to grain-size. Krumbein and Monk (1942) calculated *K* of unconsolidated sands with an empirical model of the form:

$$K = (760 d_w^2)\exp(-1.31\sigma_\varphi) \tag{2}$$

where d_w is geometric mean diameter (in weight) in millimetres; and σ_φ is standard deviation of the φ distribution function ($\varphi = -\log_2 d$, for d in mm).

The hydraulically-based Kozeny-Carmen equation is one of the most widely accepted and used empirical models. It was originally proposed by Kozeny (1927) and modified by Carmen (1937, 1956) to become the Kozeny-Carmen equation:

$$K = (\frac{\rho_w g}{\mu})\frac{\varphi^3}{(1-\varphi)^2}(\frac{d_m^2}{180}) \tag{3}$$

where ρ_w is fluid density; μ is fluid viscosity; φ is porosity; and d_m is representative grain size. The problem of this equation is that the choice of the representative grain size is critical to the successful prediction of *K*. Carrier (2003) noted that the model is not appropriate for either soil with an effective size >3 mm or for clayey soils.

Puckett *et al.* (1985) proposed a model and used the percent of clay-sized particles as the predictive variables of *K*:

$$K = 4.66 \times 10^{-3}\exp(-0.1975C) \tag{4}$$

where C is the clay-sized particles (%) in the soil sample. In addition, Jabro (1992) estimated *K* from bulk density and grain-size and developed the following model:

$$\log(K_s) = 9.59 - 0.81\log Si - 1.09\log C - 4.64Bd \tag{5}$$

where *Si* is the percentage of silt-sized particles; K_s is hydraulic conductivity (cm/h); *Bd* is the soil bulk density (g/cm^3).

The foregoing discussion shows clearly that the value of *K* significantly depends on the grain-size distribution and the percentage of the representative particles. Although many predictive and empirical models have been proposed in the past decades, models for fine media are scarce and a general correlation between *K* and the sorting condition integrating a wide range of soil is not yet available. The objective of this paper is to develop a new model which can estimate *K* of fine media quickly and easily, using unique representations of the grain-size distribution of soils. The new model was developed using the percentages of sand (*Sa*), silt (*Si*) and clay (*C*) sized particles, and the first and the second rank moment of the grain-size through the moment method as predictor variables. All the parameters are combined and used to predict *K* using multivariate nonlinear regression analysis.

2 MATERIALS AND METHODS

2.1 *In situ* permeameter test

The study area is located in the central Jiangning District of Nanjing, lying on the south bank of the downstream of Yangtze River, between 118°31′E–119°04′E and 30°38′N–32°13′N with an area about 329 km^2 (Fig. 1). This area lies over a sedimentary fluvial aquifer consisting mainly of gravel, sand, silt, and clay sediments. The climate is wet and warm, and the average annual temperature is 15.5°C and average annual precipitation is 1045 mm. Thirty investigation sites were selected and the sediments consisted largely of silt and sand, with a small amount of clay.

The falling-head standpipe infiltration test, as described by Nestingen (2007), Lu (2011) and Cheng *et al.* (2011), were conducted to estimate *K*. A stainless steel cutting ring, 15.0 cm in length and 10.0 cm in diameter, was pressed vertically into the soil to a depth of around 5–7cm, with the vegetation stratum removed before the test. Then, water was added from the top opening of the cutting ring and hydraulic conductivity head measurements were recorded over a time period and a column of soil was collected after the test. The soil moisture content also needs to be measured before and after the test. The derivation of *K* complied with the procedure proposed in Lu (2011).

2.2 Soil sampling and grain-size analysis

At each site, once the permeameter test was completed, a soil sample was collected and placed into a sampling bag and then all the samples were sent to the State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering for grain-size analysis. The grain-size analyses were conducted in laboratory using a Saturn DigiSizer, which can provide detailed information on the grain-size distribution. Compared to traditional methods, more comprehensive properties of samples can be obtained. All the operations complied with the River Sediment Grain Size Analysis Procedures (SL 42-92). According to the International Textural Grade, particle size less than 0.002 mm is grouped into clay, 0.002–0.02 mm is grouped as silt, and 0.02–2.0 mm is grouped as sand. In this paper, we recommend the moment method to calculate the feature parameter of the grain-size distribution. The first rank moment (X) is a measure of the mean diameter of the distribution and the second rank moment (δ) is a measure of the diffusion of the distribution and reflects the sorting condition of the sediment.

2.3 Regression model

A regression model was developed to predict K using the obtained parameters as the predictor variables (the % of Sa, Si and C-sized particles, and the first and the second rank moment). The regression model correlates the value of K to the predictor variables:

$$K = \alpha_1 exp(\beta_1 x_1) + \alpha_2 exp(\beta_2 x_2) + \alpha_3 exp(\beta_3 x_3) + \alpha_4 x_4^{\beta_4} + \alpha_5 x_5^{\beta_5} \tag{6}$$

where K is computed hydraulic conductivity; and α_i, β_i are coefficients determined by the regression analysis (i = 1 to 5); and x_i is the value of the predictor variables. Based on what has been discussed, it can be concluded that the predictor variables are relevant to K. Accordingly, all of them are used in the regression analysis.

The reliability and accuracy of the model can be answered by using: (1) the coefficient of determination R^2; and (2) the standard error of estimate S_e. The coefficient of determination R^2 equals the probability that the variance can be explained by the predictor variables. Thus, R^2 is an indicator of accuracy. The magnitude of S_e is a physical indicator of the error between the computed and measured values.

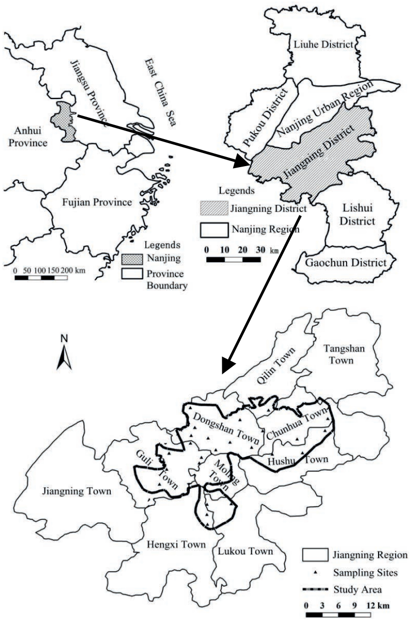

Fig. 1 Locations of the study area and the experiment sites.

3 RESULTS AND DISCUSSION

3.1 Graphical analysis

Plots of K versus the proposed variables, the percentages of sand, silt and clay-sized particles, and the first and the second rank moments were statistically analysed for increasing or decreasing trends. All the trends were estimated for significance using the F-statistic (Draper and Smith, 1981). Trends with higher significance have a higher value of F. The significance level at α_{fail} is an index of the significance of the trends. The significance increases as the value of α_{fail} decreases.

Figure 2(*a*)–(*c*) shows K respectively as a function of the percent of clay, silt and sand-sized particles of the grain-size distribution, respectively. Two different trends can be observed, in which K increases with an increase in the percent of clay and silt, while an inverse trend is observed between K and the percent of sand-sized particles. Intuitively, this conclusion is irrational. Soils having a higher quantity of fines and clay, generally have lower hydraulic conductivity. But a counter conclusion, consistent with this paper, was obtained by Craig *et al.* (1995) and Boadu (2000). A positive coefficient is present for clay content and percent of fines, in their regression model which indicates that hydraulic conductivity increases with increasing clay content and fines. Increasing clay content, while maintaining a constant plasticity index, indicates that the clay fraction is composed of less active minerals, and results in higher hydraulic conductivity (Craig *et al*., 1995). Sometimes, for coarse content, the correlation between K and the gravel content is likely to be inversely proportional on a local scale. When increasing the quantity of gravel content in sand-clay soil causes an increase in soil resistivity, it will also result in K decreasing (Shevnin, *et al.* 2006). Based on what have been discussed, the phenomenon observed in Fig. 2 can be well explained.

To confirm the significance of the trends, the values of measured K were regressed linearly on the percent of C, Si and Sa-sized particles. The F-statistics were F = 41.71, 33.50, 32.17, respectively, and α_{fail} =2.4E-8, 3.0E-7, 4.7E-7 (less than 0.01). The outcomes of the F-statistics substantiate that the trends are significant at the level of 0.05.

Plots of K as a function of the first and the second rank moment, respectively, are shown in Fig. 2(d) and (e). An inverse trend between K and the first and the second rank moments is clear. The F-statistics ($F = 7.35$, 27.64, $\alpha_{\mathrm{fail}} < 0.01$) indicate that the trend between the two variables and K is significant at the 0.05 level.

3.2 Hydraulic conductivity prediction

Although the relationship between K and the five parameters is not very strong, integrating them into a multivariate system and implementing a multivariate regression analysis may substantially improve the relations (Draper and Smith, 1981). Consequently, the multivariate regression model can increase the reliability and accuracy of the prediction. The regression model which best correlates K to the predictors is:

$$\begin{aligned} K = {} & 1.09\times 10^{-11}\exp(184.7C) + 8.31\times 10^{-9}\exp(39.33Si) + \\ & 1.61\times 10^{7}\exp(-71.18Sa) + 0.05X^{-1.11} + 0.006\delta^{-0.74} \end{aligned} \tag{7}$$

This equation describes a model which estimates K of fine media using information obtained from the grain-size distribution. The model is expressed by a functional relation between K and a set of five parameters. R^2 is 0.75, implying 75% of the variance in K can be explained by the model.

The reliability of the model was assessed by comparing outputs calculated from it with the measured values, and computing the correlation coefficient and the standard error of the estimation (Fig. 3). The correlation coefficient at 95%confidence interval is 0.87 ($R^2 = 0.75$), the standard error of estimate (S_e) is 2.13 (less than the mean value of K), and the root-mean-square error is 1.54. These indicators are statistically significant, indicating that the measured values and the predicted values are comparable. Figure 3 shows that the estimated values with multi-variations do not agree well with the *in situ* observations; this may be due to many other factors which relate to the values of K, such as the soil bulk density, the soil moisture content and the compacted

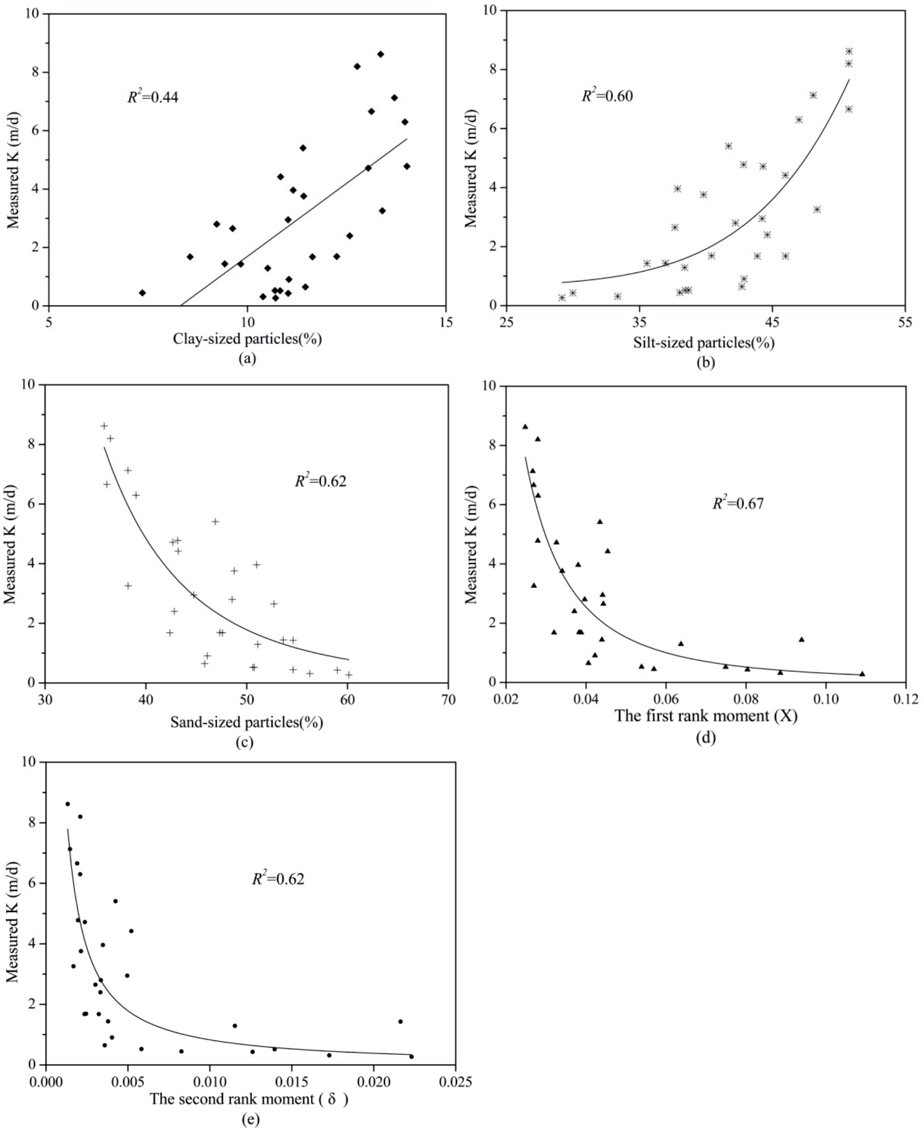

Fig. 2 Plots of the relationship between hydraulic conductivity and the five predictor variables of grain-size distribution.

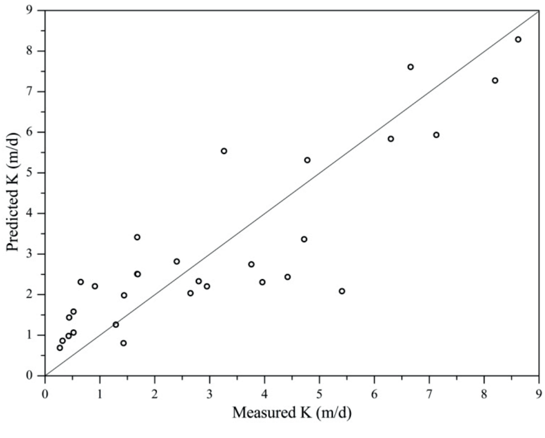

Fig. 3 Plot of measured hydraulic conductivity K_m *versus* predicted values K_p using established regression model (straight line represents line of perfect quality).

condition, etc., that were not considered in this study. If more factors are taken into account, the accuracy of the model and the fitting of the measured *K* and the predicted *K* could be improved.

4 SUMMARY AND CONCLUSIONS

According to the previous research, estimation of *K* from measured soil textural properties is a rational alternative to the field and experimental methods. In this paper, an empirical model is proposed to estimate *K* of soils from grain-size distribution. The new model was based on multivariate nonlinear regression analysis and its results are reliable and explain the whole spectrum of grain-size distribution via the percentages of sand, silt and clay-sized particles, and the first and the second rank moment of the grain-size through the moment method. The main achievement of this research is the incorporation of the first and the second rank moment of the grain-size distribution as characteristic parameters of grain-size in the new model for determining the value of *K*.

The new empirical model is deemed to be a rational substitute to experimental methods. Its superiority will be highlighted when the preparation of the field experiment is difficult. In certain cases, it may also be applied to provide first-hand information about hydraulic properties in a field environment, such as the estimated values of *K* where field operations are prohibited by some physical factors.

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