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An estimation method of the direct benefit of a waterlogging control project applicable to the changing environment

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Abstract The direct benefit of a waterlogging control project is reflected by the reduction or avoidance of waterlogging loss. Before and after the construction of a waterlogging control project, the disaster-inducing environment in the waterlogging-prone zone is generally different. In addition, the category, quantity and spatial distribution of the disaster-bearing bodies are also changed more or less. Therefore, under the changing environment, the direct benefit of a waterlogging control project should be the reduction of waterlogging losses compared to conditions with no control project. Moreover, the waterlogging losses with or without the project should be the mathematical expectations of the waterlogging losses when rainstorms of all frequencies meet various water levels in the drainage-accepting zone. So an estimation model of the direct benefit of waterlogging control is proposed. Firstly, on the basis of a Copula function, the joint distribution of the rainstorms and the water levels are established, so as to obtain their joint probability density function. Secondly, according to the two-dimensional joint probability density distribution, the dimensional domain of integration is determined, which is then divided into small domains so as to calculate the probability for each of the small domains and the difference between the average waterlogging loss with and without a waterlogging control project, called the regional benefit of waterlogging control project, under the condition that rainstorms in the waterlogging-prone zone meet the water level in the drainage-accepting zone. Finally, it calculates the weighted mean of the project benefit of all small domains, with probability as the weight, and gets the benefit of the waterlogging control project. Taking the estimation of benefit of a waterlogging control project in Yangshan County, Guangdong Province, as an example, the paper briefly explains the procedures in waterlogging control project benefit estimation. The results show that the waterlogging control benefit estimation model constructed is applicable to the changing conditions that occur in both the disaster-inducing environment of the waterlogging-prone zone and disaster-bearing bodies, considering all conditions when rainstorms of all frequencies meet different water levels in the drainage-accepting zone. Thus, the estimation method of waterlogging control benefit can reflect the actual situation more objectively, and offer a scientific basis for rational decision-making for waterlogging control projects.

Key words changing environment; waterlogging control benefit; waterlogging loss; estimation; mathematical expectation; copula function; joint probability density function

1 BACKGROUND

The term waterlogging disaster refers to a condition such that, due to excessive rainfall, surface rainwater cannot be immediately removed, resulting in surface ponding, which immerses the disaster-bearing body, so that damages are caused to the functions and values of the disaster-bearing body, and cause losses to the economy. Obviously, the degree of ultimate disaster is closely related to the condition of water accumulation and the capability of the disaster-bearing body to withstand disasters. The condition of water accumulation caused by the same rainstorm is not only related to the disaster-inducing environment in a waterlogging-prone zone (referring to the condition of the underlying surface, water storage capacity, drainage systems, etc.), but also closely related to the water level in the drainage-accepting zone. Therefore, for the same rainstorm, any change in the factors of the disaster-bearing body or disaster-inducing environment of the waterlogging-prone zone or water level in the drainage-accepting zone may lead to a corresponding change in the degree of disaster, and the waterlogging loss will change along with it.

The disaster-inducing environment of a waterlogging-prone zone may change over time, and the type, quantity and spatial distribution of a disaster-bearing body may also change over time. For instance, with urbanisation, the changes of underlying surface lead to changes in the conditions of runoff generation and flow concentration. Apart from this, urbanization is often accompanied by industrial structure adjustment and the disaster-bearing body is rapidly changing, so the total economic value and the disaster-bearing capacity will certainly change accordingly.

In addition, the drainage-accepting zone may be a river, lake or sea, of which the water level generally changes, especially when the drainage-accepting zone is a tidal river or sea. The water level has an influence over the draining-off of water from water-logged zone.

The direct economic benefit of a waterlogging control project is the loss that can be reduced or avoided by the project, which is the difference between the loss suffered without the project and the loss suffered with the project already in operation. Therefore, under the changing environment, the benefit of a waterlogging control project should be the difference between the waterlogging losses with and without the waterlogging control project. And the waterlogging loss, no matter with or without the waterlogging control project, should be the mathematical expectation value of waterlogging loss under all possible combinations of rainstorms in the waterlog-prone zone and water levels in the drainage-accepting zone. The reason is that a rainstorm, no matter what frequency it has, may meet various water levels in the drainage-accepting zone.

Methods commonly used for waterlogging control benefits estimation, such as the measurement of accumulated waterlogging, the correlation analysis method of axis, the actual annual series method (Shi Xichan and Jiang Shuixin 2005, Wang Li-ping *et al.* 2008, Zhang Dazhi. 2004) are currently not suitable for the assessment of waterlogging control benefits when the environment is changing. Up till now, there were few studies on the estimation of waterlogging control benefits (Guo qi. (2002), Li Jiren *et al.* 2003, Zhu Xuping *et al.* 2007), which never provide an actual method to estimate waterlogging control benefits. With the rapid development of urbanization and economic society, waterlogging disasters have become more and more serious. It is urgent to strengthen the waterlogging control, and thus it is critical to develop new methods to estimate waterlogging control benefits under the constantly changing environment. The key of the study is to investigate how to predict waterlogging control benefits in any year.

The estimation model of the direct benefit of waterlogging control projects put forward in this paper may be suitable for the changes of both disaster-inducing environment and disaster-bearing bodies. In addition, it considers all the combinations between rainstorms of the waterlog-prone zone and water levels in the drainage-accepting zone. Therefore, the proposed estimation method of waterlogging control benefits can objectively reflect the actual benefits. Taking the estimation of the benefit of a waterlogging control project in Yangshan County, Guangdong Province as an example, the paper briefly explains the procedures in waterlogging control project benefit estimation.

2 THE WATERLOGGING CONTROL BENEFITS ESTIMATION MODEL

2.1 The construction of the estimation model of waterlogging control benefits

Both the rainstorm in a waterlog-prone zone, H, and the water level encountered in a drainage-accepting zone, Z, are continuous random variables. For any possible combination of h and z, referred to as (h, z), both the waterlogging loss without a waterlogging control project, S_0, and the loss with a control project, S_1, are functions of h. Assuming that $S_j = g_j(h, z)(j = 0,1)$, where $g_0(h, z)$ and $g_1(h, z)$ are the waterlogging loss functions with or without a waterlogging control project, respectively. Then, S_j, i.e. S_0 and S_1, is also a random variable. The waterlogging loss with or without a control project can be generally represented as $E(S_j)(j = 0,1)$. Assuming that the probability density of two-dimensional random variable (H, Z) related to the point (h, z), i.e. the combination of h and z, is $f(h, z)$. Then:

$$E\left(S_j\right) = E\left[g_j\left(h,z\right)\right] = \int_0^{+\infty}\int_{z_0}^{+\infty} g_j\left(h,z\right) f\left(h,z\right)\mathrm{d}h\mathrm{d}z = \iint_D g_j\left(h,z\right) f\left(h,z\right)\mathrm{d}h\mathrm{d}z \tag{1}$$

where D is the integral region, D: $0 \le h < +\infty$, $z_0 \le z < +\infty$. z_0 is the possible minimal water level in a drainage-accepting zone. The integral region can be further determined by combining H (the rainstorm in a waterlog-prone zone) and Z (the corresponding water level in a drainage-accepting zone) with the marginal probability density distribution $f_H(h)$ and $f_Z(z)$.

Therefore, the waterlogging control benefit, B, is:

$$B = E(S_0) - E(S_1) = \iint_D [g_0(h,z) - g_1(h,z)] f(h,z) \mathrm{d}h\mathrm{d}z \tag{2}$$

where $f(h, z)$ is the probability density of H and Z at the point (h, z), i.e. the combination of h and z.

$f(h, z)$ can be obtained according to the methods introduced by Liu Zengmei and Chen Zhishen. (2009, 2011). This makes it possible to apply the method of waterlogging control benefits calculation mentioned above. However, it is still difficult to use equation (1) to directly estimate the expectation value of waterlogging loss with or without a control project. First of all, $S_j = g_j(h, z)(j = 0,1)$ is generally not available. Secondly, two integrals may not be given by the elementary function. Hence, it is necessary to develop practical calculation methods.

Here, the widely used "segmentation-approximate-integration" method in engineering mathematics is applied to approximately solve equation (1). Its calculation precision is closely related to how fine the segmentation of the integral region D is. The finer, the accuracy of the results is higher.

Two families of straight lines, which are respectively parallel with two axes and have individual integral distance of Δh and Δz, divide the integral region D into many small closed areas $\Delta\sigma_i$ (i =1,2…,n) (Fig. 1). Δh and Δz do not need to be fixed values. Then:

$$E(S_j) = E[g_j(h,z)] = \iint_D g_j(h,z) f(h,z)\mathrm{d}h\mathrm{d}z \approx \sum_{i=1}^{n} \iint_{\Delta\sigma_i} g_j(h,z) f(h,z)\mathrm{d}h\mathrm{d}z \tag{3}$$

The average waterlogging loss $\overline{s_{ji}}$ at $\Delta\sigma_i$ is expressed as (j = 0,1; i =1,2……n). $\iint_{\Delta\sigma_i} f(h,z)\mathrm{d}h\mathrm{d}z$ is the probability, Δp_i, of the point (h, z) at $\Delta\sigma_i$. The approximate calculation equation is obtained by equation (4).

$$E(S_j) = E[g_j(h,z)] \approx \sum_{i=1}^{n} \iint_{\Delta\sigma_i} g_j(h,z) f(h,z)\mathrm{d}h\mathrm{d}z \approx \sum_{i=1}^{n} \overline{s_{ji}} \iint_{\Delta\sigma_i} f(h,z)\mathrm{d}h\mathrm{d}z = \sum_{i=1}^{n} \overline{s_{ji}} \Delta p_i \tag{4}$$

Hence, the equation of waterlogging control project benefits is as follows:

$$B = E(S_0) - E(S_1) \approx \sum_{i=1}^{n} \overline{s_{0i}} \Delta p_i - \sum_{i=1}^{n} \overline{s_{1i}} \Delta p_i = \sum_{i=1}^{n} \left(\overline{s_{0i}} - \overline{s_{1i}}\right) \Delta p_i = \sum_{i=1}^{n} \Delta B_i \Delta p_i \tag{5}$$

Equation (5) shows that the probability is used as weight to perform the weighted average for the waterlogging control benefits of all small rectangular regions. Its average is approximate to the waterlogging control benefit of the project.

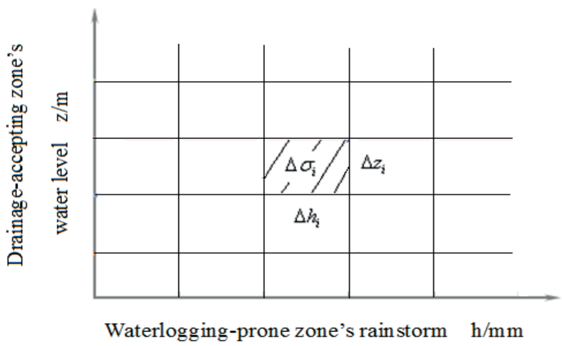

Fig. 1 Sketch map of division of integral area.

The length and width of each small rectangle region $\Delta\sigma_i$ are Δh_i and Δz_i, respectively. If the coordinate of the rectangle's lower left corner is (h_i, z_i), the coordinates of the other three corners are $(h_i+\Delta h_i, z_i)$, $(h_i, z_i+\Delta z_i)$ and $(h_i+\Delta h_i, z_i+\Delta z_i)$. If the joint distribution of the rainfall in a waterlog-prone zone, H, and the water level in a drainage-accepting zone, Z, is expressed as $F(h, z)$. Then:

$$\Delta p_i = \iint_{\Delta\sigma_i} f(h,z)\mathrm{d}h\mathrm{d}z = F(h,z)\big|_{\Delta\sigma_i} = F(h_i + \Delta h_i, z_i + \Delta z_i) - F(h_i, z_i + \Delta z_i) - F(h_i + \Delta h_i, z_i) + F(h_i, z_i) \tag{6}$$

The average waterlogging loss, $\overline{s_{ji}}$, at $\Delta\sigma_i$ can be calculated with the following equation:

$$\overline{s_{ji}} = g_j\left(h_i + \frac{1}{2}\Delta h_i, z_i + \frac{1}{2}\Delta z_i\right) \tag{7}$$

Therefore, the key of waterlogging control benefit estimation is to calculate the join distribution, $F(h_i, z_i)$ and the probability density, $f(h_i, z_i)$, and the waterlogging loss, $g_i(h_i, z_i)$.

2.2 The steps of waterlogging control benefit estimation

According to the analysis above, the waterlogging control benefit of a control project can be calculated in following steps:

(1) Collect and organize data and obtain the necessary relation curve in calculating waterlogging loss, combining with analysis and calculation.
(2) Collect data about the annual maximum rainstorms of the waterlog-prone zone and the water levels in the drainage-accepting zone, and analyse and determine the marginal distribution, the joint distribution and joint probability density function of the two variables.
(3) Determine the integral region according to the annual maximum rainstorms, marginal distribution of the corresponding water levels in the drainage-accepting zone and their joint probability density distribution. Rationally divide the integral region.
(4) Calculate the individual probability, Δp_i, of each segmented small rectangular region, and obtain the region's difference value of average waterlogging loss $\overline{s_{1i}}$, with a waterlogging control project and average loss $\overline{s_{0i}}$, without a control project. The difference value is the region's waterlogging control benefit, ΔB.
(5) Calculate the average waterlogging control benefit, $B \approx \sum_{i=1}^{n} \Delta B_i \Delta p_i$, i.e. the weighted average is obtained of all the waterlogging control benefits of all the small rectangle areas, using the probability as the weight. The average is approximate to the waterlogging control benefit of the waterlogging control project

3 CASE ANALYSIS

A waterlog-prone zone in the town of Yangshan of Guangdong Province was used as an example to briefly illustrate the analysis process of the waterlogging control benefit. In recent years, the waterlog-prone zone has developed so rapidly that original vegetable farming land has been changed to urban land. We only calculated the waterlogging loss and waterlogging control benefit in the first year (2010) after the waterlogging control project construction.

First, we obtained the spatial distribution of the various disaster-bearing bodies or diverse assets in the waterlogging-prone zone and got the relation of the submerging depth with the waterlogging loss rate, which uses submerging time as a parameter.

Table 1 shows the estimated values of various classified assets in the waterlogging-prone zone in 2010, simplified to the average distribution, and also includes the 2010 average waterlogging loss rate of each classified asset under different submerging depth.

Then, we analysed the marginal distribution and joint distribution of the variables, the rainstorm in the waterlog-prone zone and the water level in the drainage-accepting zone, to obtain the joint probability density function of the two variables.

The marginal distribution and marginal probability density function of the annual maximum daily rainstorm, H, are $F_H(h)$ and $f_H(h)$, respectively, and of the corresponding daily average water level, Z, are $F_z(z)$ and $f_z(z)$, respectively. We found that a P-III type distribution fits both the marginal distribution $F_H(h)$, and the corresponding daily average flood level, $F_z(z)$. The statistical parameters were $\bar{h}$= 97 mm, C_v = 0.38, C_s = 1.33 and $\bar{z}$ = 60.57 m, C_v = 0.024, and C_s = 0.63, respectively. $f_H(h)$ and $f_z(z)$ are respectively shown as follows:

Table 1 The value of each asset and the waterlog loss percentage in the case of different submerge in 2010.

Asset classification	All kinds of asset value/100 Yuan	The social-asset loss ratio under different submerging depth (%)			
		0~0.3m	0.3~0.6m	0.6~0.9m	0.9~1.2m
Fixed assets of industry and commerce	0.36	2.5	5	5	5
Flow assets of industry and commerce	0.10	2.5	6	8	10
Business inventory goods and materials	0.09	1	2	3	4
Assets of construction industry	0.15	0.5	1	2	3
The communication system	0.05	0.5	1	2	3
Residential property	0.18	1.5	4	8	10
Residential housing	0.37	1	4	8	10
Foundation engineering like traffic, water supply, etc.	0.57	0	1.5	3	5
Individual enterprises	0.001	2.5	5	5	5

$$f_H(h) = \frac{0.0408^{2.2613}}{\Gamma(2.2613)}(h-41.57)^{1.2613}e^{-0.0408(h-41.57)}\ ,\quad h \geq 41.57 \tag{9}$$

$$f_Z(z) = \frac{2.1838^{10.0781}}{\Gamma(10.0781)}(z-55.96)^{9.0781}e^{-2.1838(z-55.96)}\ ,\quad z \geq 55.96 \tag{10}$$

The Clayton Copula function described the correlation structure between the maximum rainstorm and corresponding daily average water level better than a Gumbel Copula function. In this case, θ = 10.5. Consequently, we got the joint distribution, F (h, z), and the joint probability density function, f (h, z), respectively expressed as follows:

$$F(h,z) = \left\{[F_H(h)]^{-10.5} + [F_Z(z)]^{-10.5} - 1\right\}^{-1/10.5} \tag{11}$$

$$f(h,z) = 11.5\left[F_H(h)F_Z(z)\right]^{-11.5}\left\{\left[F_H(h)\right]^{-10.5} + \left[F_Z(z)\right]^{-10.5} - 1\right\}^{\frac{-1}{10.5}-2} f_H(h)f_Z(z) \tag{12}$$

Based on the joint probability density distribution (Fig. 2) and the marginal distribution, the integral area was defined as (41.57~200 mm, 55.96~64.5 m). The waterlogging design combination of the waterlogging-prone zone was between the 1-in-10 years rainstorm (146 mm) and the 1-in-5 years annual highest daily average water level (62.21 m) in the drainage-accepting zone. Hence, the integral region was segmented as shown in Fig. 3, integrating the waterlogging control design combination.

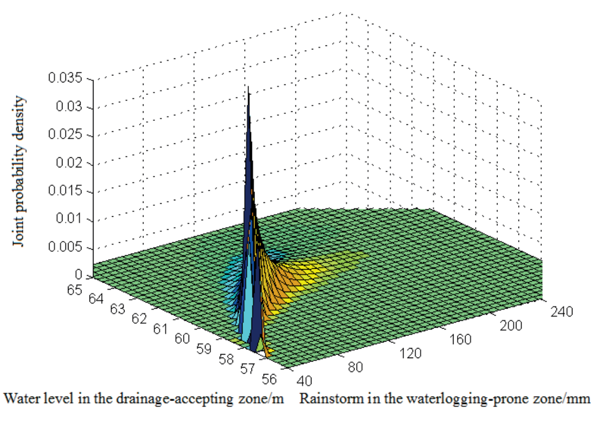

Fig. 2 The joint probability density distribution maps of the two variables.

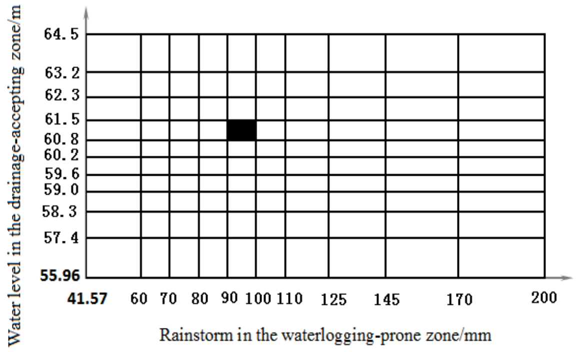

Fig. 3 The map of division of integral area.

Then, each region was calculated, and the region in Fig. 3 was used as an example to illuminate the analysis.

The midpoint coordinates of the area were (95, 61.15), suggesting the rainstorm of the waterlogging-prone zone was 95 mm and the corresponding water level was 61.15 m. Because the design rainstorm and the waterlog control design water level were 146 mm and 62.21 m, respectively, the area had no waterlogging loss with the waterlogging control project. So, the water-logging control benefit of this region was expressed as: $\Delta B_i = \overline{s_{0i}} - \overline{s_{1i}} = 163.64 - 0 = 1.64$ million.

The four vertex coordinates of this area were (90, 60.8), (100, 60.8), (100, 61.5) and (90, 61.5), respectively. So the frequency of the area was $\Delta p_i = F(100, 61.5) - F(100, 60.8) - F(90, 61.5) + F(90, 60.8) = 0.6127 - 0.5708 - 0.5102 + 0.5030 = 0.0347$.

Similarly, the waterlogging control benefit, ΔB_i, and probability weight, Δp_i, of each area could be obtained, and therefore the 2010 waterlogging control benefit was calculated as $B = \sum \Delta B_i \Delta p_i = 0.695$ (million Yuan).

4 CONCLUSION

None of the existing estimation methods of the direct benefit of waterlogging control projects is suitable for the changing environment. The estimation model of waterlogging control benefits proposed in this paper overcomes this defect. It is suitable for the changes of both disaster-inducing environment and disaster-bearing bodies. In addition, it considers all the combinations between rainstorms of the waterlogging-prone zone and water levels in the drainage-accepting zone. Therefore, the proposed estimation method of waterlogging control benefits can objectively reflect the actual benefits, and thus provides a scientific basis for the rational decisions on waterlogging control project construction. But the application of the model will usually be restricted due to basic data deficiencies.

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